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Question Paper Code : 80212

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2019.

Third Semester

Computer Science and Engineering

MA 8351 — DISCRETE MATHEMATICS

(Common to Information Technology)

(Regulation 2017)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Write the inverse of the statement, "If you work hard then you will be rewarded".

2. If the universe of discourse consists of all real numbers and if $p(x)$ and $q(x)$ are given by $p(x): x \geq 0$ and $q(x): x^2 \geq 0$, then determine the truth value of $(\forall x)(p(x) \to q(x))$.

3. Prove that if n and k are positive integers with $n = 2k$, then $\frac{n!}{2^k}$ is an integer.

4. How many solutions does the equation, $x_1 + x_2 + x_3 = 11$ have, where x_1, x_2 and x_3 are non-negative integers?

5. If G is a simple graph with $\delta(G) \geq \frac{|V(G)|}{2}$ then show that G is connected.

6. Give an example of a graph which is Hamiltonian but not Eulerian.

7. Is it true that $(\mathbb{Z}_5^*, \times_5)$ a cyclic group? Justify your answer.

8. Prove that group homomorphism preserves identity.

9. Show that in a lattice if $a \leq b$ and $c \leq d$ then $a * c \leq b * d$.

10. Is it true that every chain with at least three elements is always a complemented lattice? Justify your answer.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Obtain the principal conjective normal form of the formula $(\urcorner P \to R) \wedge (P \to Q) \wedge (Q \to P)$. (6)

(ii) Using indirect method, show that $R \to \urcorner Q$, $R \vee S$, $S \to \urcorner Q$, $P \to Q \Rightarrow \urcorner P$. (10)

Or

(b) (i) Show that the premises "A student in this class has not read the book" and "Everyone in this class passed the Semester Exam" imply the conclusion "Someone who passed the Semester Exam has not read the book". (10)

(ii) Prove that $(\exists x)(P(x) \wedge Q(x)) \Rightarrow (\exists x) P(x) \wedge (\exists x) Q(x)$. (6)

12. (a) (i) Let $m \in Z^{+}$ with m odd. Then prove that there exists a positive integer n such that m divides $2^{n} - 1$. (6)

(ii) Determine the number of positive integers n, $1 \leq n \leq 2000$ that are not divisible by 2, 3 or 5, but are divisible by 7. (10)

Or

(b) (i) Using mathematical induction, prove that every integer $n \geq 2$ is either a prime number or product of prime numbers. (6)

(ii) Using generating function method solve the recurrence relation, $a_{n+2} - 2a_{n+1} + a_n = 2^n$, where $n \geq 0$, $a_0 = 2$ and $a_1 = 1$. (10)

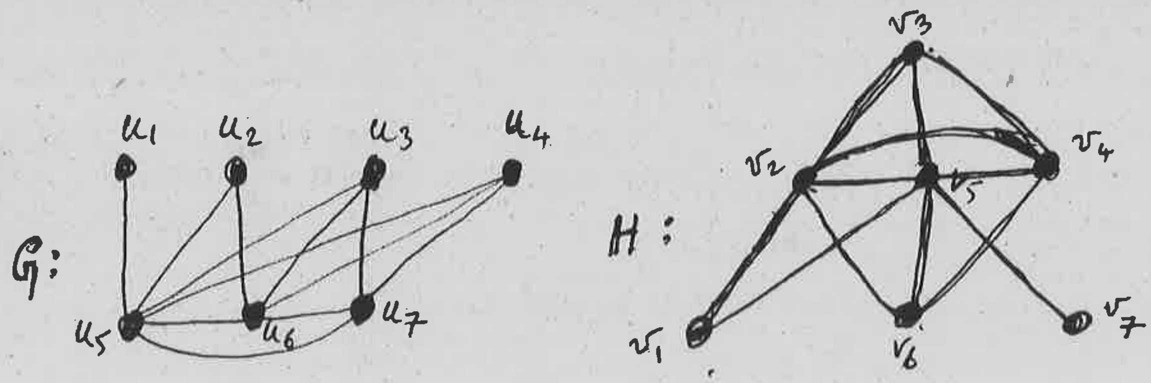
13. (a) (i) Let G be a graph with adjacency matrix A with respect to the ordering of vertices $v_1, v_2, v_3, \ldots, v_n$. Then prove that the number of different walks of length r from v_i to v_j, where r is a positive integer, equals to $(i, j)^{th}$ entry of A^r. (8)

(ii) Show that the complete bipartite graph $K_{m,n}$, with $m, n \geq 2$ is Hamiltonian if and only if $m = n$. Also show that the complete graph K_n is Hamiltonian for all $n \geq 3$. (8)

Or

(b) (i) Define incidence matrix of a graph. Using the incidence matrix of a graph G, show that the sum of the degrees of vertices of a graph G is equal to twice the number of edges of G. (6)

(ii) When do we say two simple graphs are isomorphic? Check whether the following two graphs are isomorphic or not. Justify your answer. (10)



14. (a) (i) Prove that every subgroup of a cyclic group is cyclic. (6)

(ii) Prove that every finite group of order n is isomorphic to a permutation group of degree n. (10)

Or

(b) (i) Define monoid. Give an example of a semigroup that is not a monoid. Further prove that for any commutative monoid $(M, *)$, the set of idempotent elements of M form a submonoid. (8)

(ii) Let $(G, *)$ be a group and let H be a normal subgroup of G. If G/H be the set $\{aH \mid a \in G\}$ then show that $(G/H, \otimes)$ is a group, where $aH \otimes bH = (a * b)H$, for all $aH, bH \in G \mid H$. Further, show that there exists a natural homomorphism $f : G \to G \mid H$. (8)

15. (a) (i) If (A, R) is a partially ordered set then show that the set (A, R^{-1}) is also a partially ordered set, where $R^{-1} = \{(b, a) / (a, b) \in R\}$. (6)

(ii) Let $(L, *, \oplus)$ and $(M, \wedge, \vee)$ be two lattices. Then prove that $(L \times M, \Delta, \nabla)$ is a lattice, where $(x, y) \Delta (a, b) = (x * a, y \wedge b)$ and $(x, y) \nabla (a, b) = (x \oplus a, y \vee b)$, for all $(x, y), (a, b) \in L \times M$. (10)

Or

(b) (i) Prove that in every lattice distributive inequalities are true. (8)

(ii) Define modular lattice. Prove that a lattice L is modular if and only if $x, y \in L$, $x \oplus (y * (x \oplus z)) = (x \oplus y) * (x \oplus z)$. (8)
